Chapter 3 - Factors and Products: Skills Summary

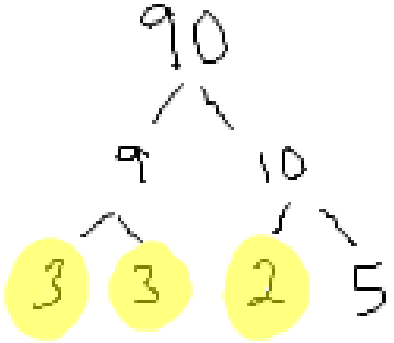
1. **Skill: Determine prime factors, greatest common factor (GCF) and least common multiple (LCM).**

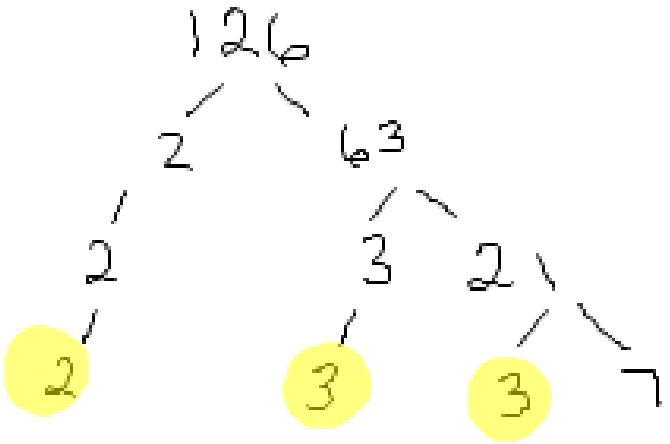
Strategy: 1. Create a factor tree of prime factors (# which has only the factors one and itself)

2. GCF = product of common prime factors
3. LCM = product of highest power of each prime factor

Example: → "Go fish"

a) Find the greatest common factor of 90 and 126.



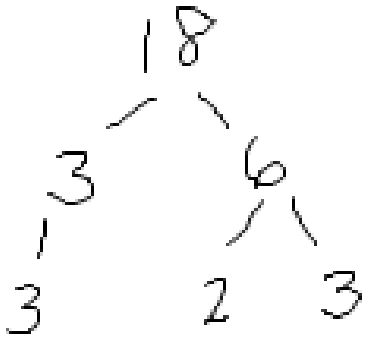


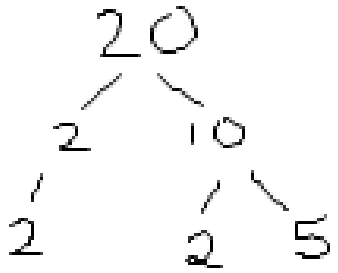
GCF = 3 · 3 · 2

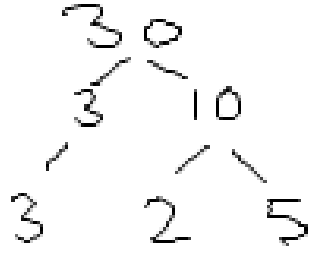
= 18

GCF = 18

b) Find the lowest common multiple of 18, 20, 30.







Write each as product of prime factors

$3^2 \cdot 2$ $\quad 2^2 \cdot 5$ $\quad 3 \cdot 2 \cdot 5$

Take one of each base, but has to be one with largest exponent

$LCM = 3^2 \cdot 2^2 \cdot 5$

$= 9 \cdot 4 \cdot 5$

$LCM = 180$

2. Skill: Determine whether a number is a perfect square or a perfect cube using factorization.

Strategy: Make a factor tree and determine whether factors occur in pairs or sets of three.

Example:

Determine the ~~square~~ cube root of 216.

Show Work

$\sqrt[3]{216}$

$= \sqrt[3]{4 \cdot 54}$

$= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}$

$= 2 \cdot 3$

$= 6$

Calculator

216 $\sqrt[x]{\square}$ (1 ÷ 3)

$= 6$

3. Skill: Determine common factors for a polynomial

Strategy: Look at the terms and determine their greatest common factor. Multiply the factors to check your answer.

Example: Factor : $3x^2y - 21xy + 30y^2$

$3y \rightarrow$ common to all

$3y(x^2 - 7x + 10y)$

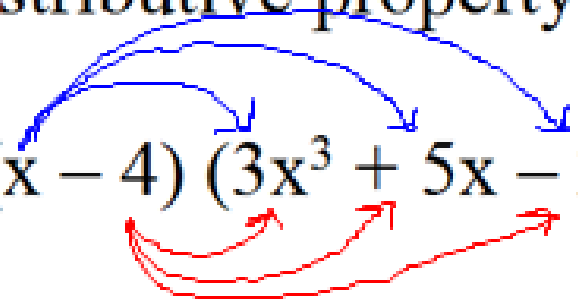
→ Can I divide all terms by "if"?

* Also, if ques. says factor you may need to common factor first but that does NOT mean you're done.

4. Skill: Multiply/Expand binomials

Strategy: distributive property OR use algebra tiles

Example: $(x-4)(3x^3+5x-2)$



multiply each term in first bracket by each term in second bracket

Distributive property

$= 3x^4 + 5x^2 - 2x - 12x^3 - 20x + 8$

organize & combine like terms

$= 3x^4 - 12x^3 + 5x^2 - 22x + 8$

5. Skill: Factor polynomials in the form $x^2 + bx + c$

Strategy: use algebra tiles OR find factors of c that add to give you b

$ax^2 \quad bx \quad c$

find two #s that multiply to make -12 → (+)(−)

and add to make $+4$

Example: Factor $x^2 + 4x - 12$

$(x + 6)(x - 2)$

-12 →

$-1 \quad 12$

$+1 \quad -12$

$+3 \quad -4$

$+4 \quad -3$

$+6 \quad -2$

$+2 \quad -6$

6. Skill: Factor polynomials in the form $ax^2 + bx + c$

Strategy: factor by decomposition

$ax^2 \quad bx \quad c$

Example: Factor $5x^2 - 9x - 2$

$ac = 5 \cdot -2 = -10$

factors of -10 that add to make -9 (b)

$-10 \cdot 1$

Then break into "chunks" and common factor

$-9x$

$5x^2 - 10x + 1x - 2$

$5x(x-2) + 1(x-2)$

$= (x-2)(5x+1)$

7. Skill: Factor special polynomials

Strategy: Recognize trinomials that are perfect squares or difference of squares (might need to divide out common factors first).

$25x^2 + (-49y^2)$

two perfect squares but no middle term → difference of squares

Example: a) Factor $25x^2 - 49y^2$

$(5x - 7y)(5x + 7y)$

* When you have a difference of squares the first term and second term are pefect squares, there is no middle term and terms must be separated by subtraction

b) Factor $4x^2 + 16xy + 16y^2$

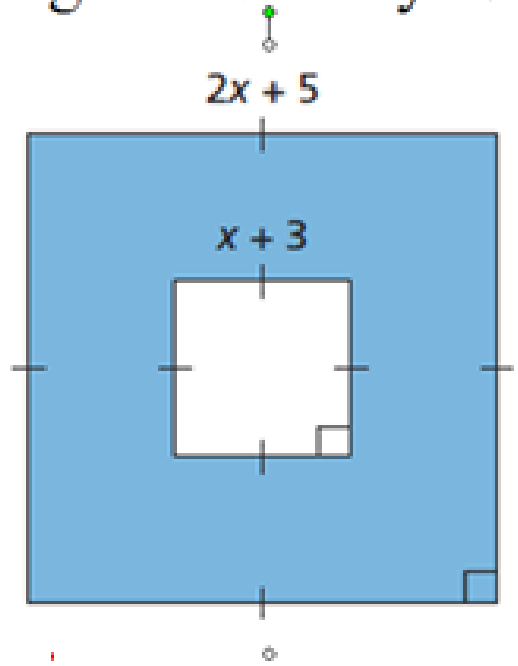
$4(x^2 + 4xy + 4y^2)$

$4(x + 2y)(x + 2y)$

$= 4(x + 2y)^2$

8. **Skill: Solve problems involving factors and products.**

Example: Determine the area of the shaded region. Write your answer in simplest form.



Areas

figure out big box and subtract the little box

big box little box

$A = (2x+5)(2x+5) - (x+3)(x+3)$

Distributive property for each piece

$A = 4x^2 + 10x + 10x + 25 - [x^2 + 6x + 9]$

$A = 4x^2 + 20x + 25 - x^2 - 6x - 9$

$$\boxed{A = 3x^2 + 14x + 16}$$

Factoring Review:

How do I know which method to use????

1. Before factoring, try to find a common factor of all terms to factor out.
2. Rearrange your polynomial and factor out -1 if needed so it is in the form $ax^2 + bx + c$.
3. Then look to see if you can factor without decomposition. (a = 1)
4. Use decomposition only if needed (a > 1)

a) $x^2 + 6x$

b) $18 + 9x^2$

c) $50 - 15m + m^2$

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d) $2y^2 + 5y + 2$

e) $2n^2 + 9n - 18$

f) $4k^2 - 4k - 80$

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g) $15e^2 - 7e - 2$