Chapter 3 - Factors and Products: Skills Summary

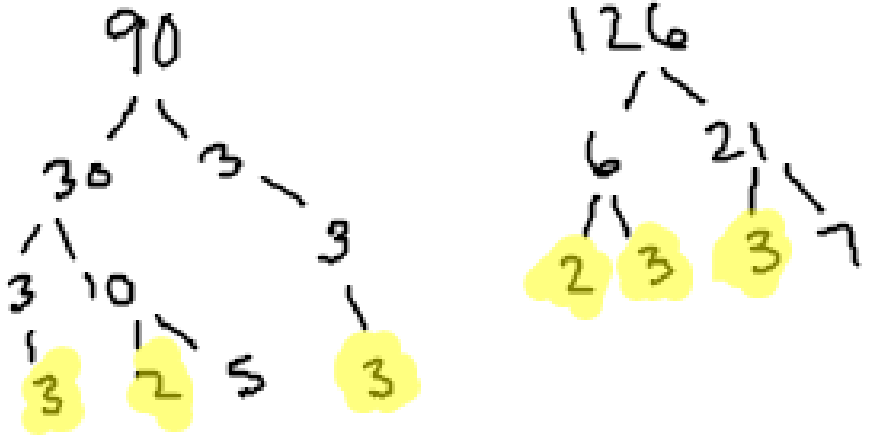
1. **Skill: Determine prime factors, greatest common factor (GCF) and least common multiple (LCM).**
   Strategy: 1. Create a factor tree of prime factors (# which has only the factors one and itself)
   * 2. GCF = product of common prime factors
   3. LCM = product of highest power of each prime factor

Example:

a) Find the greatest common factor of 90 and 126.

GCF

↓ "I Go fish"

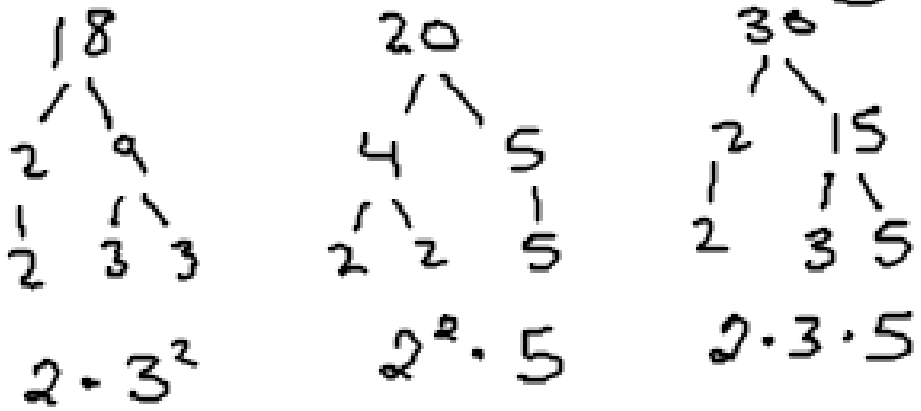


GCF : 3 · 2 · 3

= 18

b) Find the lowest common multiple of 18, 20, 30.

LCM → like skip Counting



$2 \cdot 3^2$ $2^2 \cdot 5$ $2 \cdot 3 \cdot 5$

LCM → each base has to be represented but need each with highest exponent

LCM = $2^2 \cdot 3^2 \cdot 5 = 180$

2. Skill: Determine whether a number is a perfect square or a perfect cube using factorization.

Strategy: Make a factor tree and determine whether factors occur in pairs or sets of three.

Example:
Determine the square root of 216.

cubed

groups of 3

$\sqrt[3]{216}$

216
/ \

$= \sqrt[3]{2 \cdot 108}$

$= \sqrt[3]{2 \cdot 4 \cdot 27}$

$= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 9 \cdot 3}$

$= \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}$

$= 2 \cdot 3$

$= 6$

calculator

* $216 \wedge (1 \div 3)$ *

$216^{1/3}$

$\sqrt[6]{92} = 92^{1/6}$

$\sqrt[41]{31} = 31^{1/41}$

3. Skill: Determine common factors for a polynomial

Strategy: Look at the terms and determine their greatest common factor. Multiply the factors to check your answer.

Example: Factor : $3x^2y - 21xy + 30y^2$

* ALWAYS look for common factors first.

$3y(x^2 - 7x + 10y)$

$ax^2 + bx + c$

$3x^2y - 21xy + 30y$

$3y(x^2 - 7x + 10)$ *

this example still factorable

$a \cdot c$'s mult to make 10

Add -7

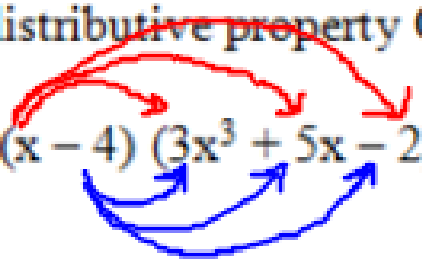
$-2 \quad -5$

$3y(x - 2)(x - 5)$

4. Skill: Multiply/Expand binomials

Strategy: distributive property OR use algebra tiles

Example: $(x-4)(3x^3+5x-2)$



* Keep an eye on neg / pos!

$= 3x^4 + 5x^2 - 2x - 12x^3 - 20x + 8$

$= 3x^4 - 12x^3 + 5x^2 - 22x + 8$

5. Skill: Factor polynomials in the form $x^2 + bx + c$

Strategy: use algebra tiles OR find factors of c that add to give you b

Example: Factor $x^2 + 4x - 12$

$(x + 6)(x - 2)$

Add to make 4	mult to make -12
+6	-2

6. Skill: Factor polynomials in the form $ax^2 + bx + c$

Strategy: factor by decomposition

Example: Factor $5x^2 - 9x - 2$ ($a = 5$, $b = -9$, $c = -2$)

Add to make -9	multiply to make $ac = 5 \cdot -2 = -10$
-10	$+1$

$= -9x$

$5x^2 - 10x + 1x - 2$

common factor

$5x(x-2) + 1(x-2)$

$= (x-2)(5x+1)$

7. Skill: Factor special polynomials

Strategy: Recognize trinomials that are perfect squares or difference of squares (might need to divide out common factors first).

Example: a) Factor $25x^2 - 49y^2$ b) Factor $4x^2 + 16xy + 16y^2$

Ans → $(5x+7y)(5x-7y)$

$25x^2 \underbrace{-35xy + 35xy}_{=0} - 49y^2$

difference of squares

No middle term needs to be subtracted

first term & 2nd term are perfect squares

$(25x^2 - 49)$

→ $4(x^2 + 4xy + 4y^2)$

$= 4(x+2y)(x+2y)$

$= 4(x+2y)^2$

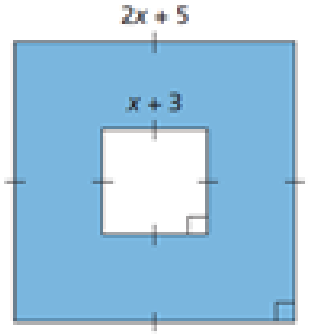
perfect Sq.

$A = l \cdot w$

8. Skill: Solve problems involving factors and products.
Example: Determine the area of the shaded region. Write your answer in simplest form.

$A = \text{Area}_{\text{big}} - \text{Area}_{\text{small}}$

$= (2x+5)(2x+5) - (x+3)(x+3)$



$= 4x^2 + 10x + 10x + 25 - [\,x^2 + 3x + 3x + 9\,]$

$= 4x^2 + 20x + 25 - x^2 - 6x - 9$

$= 3x^2 + 14x + 16$

Factoring Polynomials *How do I know which method to use????*

1. Factor out the GCF if possible.
2. Where possible, rearrange your polynomial and factor out 'a' so it is in the form $ax^2 + bx + c$.
3. Look to see if you can factor without decomposition: a = 1 OR a special case (perfect square or difference of squares)
4. Use decomposition only if needed ($a \neq 1$)

Examples:

a) $x^2 - 6x - 16$ (a = 1; b; c)

What multiplies to give c and adds to give b?

-16: -8, +2

$(x \_\_)(x \_\_)$

$(x-8)(x+2)$

b) $5x^2 - 9x - 2$ ($a \neq 1$ Decomp!)

-10: -10, 1

$5x^2 - 10x + 1x - 2$

$5x(x-2) + 1(x-2)$

$(5x+1)(x-2)$

c) $25x^2 - 49y^2$

both perf. squares; diff of squares

$(5x+7y)(5x-7y)$

d) $9b^2 + 48b + 64$

both perf. squares

$(3b+8)(3b+8)$

Factor:

a) $x^2 + 6x + 8$

b) $b^2 - b - 20$

c) $3b^2 - 13b - 10$

d) $4x^2 - 5x - 6$

e) $4x^2 + 12x + 9$

f) $36x^2 - 25$